

The axial form factor and polarization of the final nucleon in quasi-elastic $\nu - N$ scattering ¹

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Abstract

We have calculated the polarization of the final nucleon in charged current quasi-elastic $\nu - N$ scattering. We show that the longitudinal and transverse polarizations, as well as their ratio exhibit simple dependence on the axial form factor and their sensitivity to the axial mass is much stronger than that of the cross section. This suggests that measurements of the polarization of the nucleon in the high-statistics neutrino experiments could provide important information on the axial form factor.

1 Introduction

Important for understanding the electromagnetic structure of the nucleon are the two, Dirac and Pauli, electromagnetic form factors (FFs) $F_1(Q^2)$ and $F_2(Q^2)$, that determine elastic electron-nucleon scattering.

There are two ways of extracting $F_{1,2}$, or the more convenient experimentally charge and magnetic FFs $G_E = F_1 - \tau F_2$ and $G_M = F_1 + F_2$, $\tau = Q^2/4M^2$. The standard, Rosenbluth, procedure is based on the unpolarized cross section and determines G_E and G_M separately with a limited sensitivity to G_E^p at higher Q^2 . It was found:

1. At relatively low $Q^2 \lesssim 5 \text{ GeV}^2$ the proton and neutron magnetic FFs exhibit approximately the *same* dipole Q^2 -dependence, $G_D(Q^2) = (1 + Q^2/0.71 \text{ GeV}^2)^{-2}$:

$$G_M^p = \mu_p G_D(Q^2), \quad G_M^n = \mu_n G_D(Q^2). \quad (1)$$

2. Up to $Q^2 \lesssim 6 \text{ GeV}^2$ all data exhibit "scaling" of the electric G_E^p and magnetic G_M^p FFs of the proton:

$$\mathcal{R}(Q^2) = \frac{\mu_p G_E^p(Q^2)}{G_M^p(Q^2)} \simeq 1. \quad (2)$$

In the late 90-ies Jefferson Lab. started series of new type of experiments that allowed a direct measurement of \mathcal{R} . In 1968 it was shown [1] that the ratio of the transverse P_\perp and longitudinal P_\parallel polarization of the recoil proton is directly proportional to \mathcal{R} :

$$\frac{P_\perp}{P_\parallel} = - \frac{G_E^p}{G_M^p} \sqrt{\frac{2\varepsilon}{\tau(1+\varepsilon)}}, \quad (3)$$

where $\varepsilon = [1 + 2(1 + \tau)\tan^2\theta/2]^{-1}$, θ is the scattering angle. JLab measured the ratio P_\perp/P_\parallel in the energy range $Q^2=[0.5 - 8.5] \text{ GeV}^2$ and unexpected results were obtained [2]:

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1. "Scaling" *does not* hold. The form factor $G_E^p(Q^2)$ decreases much faster than $G_M^p(Q^2)$: $\mathcal{R} = 1$ at $Q^2 \simeq 1 \text{ GeV}^2$ and falls down to $\mathcal{R} = 0.2$ at $Q^2 = 5.6 \text{ GeV}^2$.
2. There is a clear discrepancy between the two methods in extracting \mathcal{R} .

Polarization experiments drastically changed our knowledge about the e.m. FFs and raised the important questions about radiative corrections and 2-photon exchange.

The JLab results strongly motivated our studies of the recoil nucleon polarization in charged current quasi-elastic (CCQE) $\nu(\bar{\nu}) - N$ scattering as a source of independent information about the axial form factor. We obtain analytic expressions for the polarization and estimate numerically the sensitivity of the polarization and the cross sections to the axial mass. Most of the presented results can be found in more details in [3].

2 The weak charged current form factors

We study the CCQE processes:

$$\nu + p \rightarrow \mu^+ + n, \quad \bar{\nu} + n \rightarrow \mu^- + p \quad (4)$$

which are the dominant processes at low neutrino energies and give a direct information on the charged current (CC) weak form factors.

The matrix elements of (4) is determined by the 4 weak CC FFs – $F_{1,2}^{CC}$, G_A and G_P :

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} (\bar{u}_\mu \gamma^m (1 \pm \gamma_5) u_\nu) \cdot \langle N' | J_\mu^{CC} | N \rangle \quad (5)$$

$$\langle N' | J_\mu^{CC} | N \rangle = \bar{u}_{N'} \left(\gamma_\mu F_1^{CC} + \frac{i\sigma_{\mu\nu} q^\nu}{2M} F_2^{CC} + \gamma_\mu \gamma_5 G_A + \frac{q_\mu}{2M} \gamma_5 G_P \right) u_N \quad (6)$$

Due to CVC $F_{1,2}^{CC}$ are related to the e.m. FFs:

$$F_{1,2}^{CC}(Q^2) = F_{1,2}^p(Q^2) - F_{1,2}^n(Q^2), \quad (7)$$

where $F_{1,2}^p$ and $F_{1,2}^n$ are the Dirac and Pauli form factors of the proton and neutron, known at present in a wide region of Q^2 [2]. The hypothesis for partial conservation of the axial current (PCAC) implies that the contribution of $G_P(Q^2)$ can be neglected. Thus, study of the CCQE processes (4) will give information about the axial form factor $G_A(Q^2)$.

In analogy with the electromagnetic FFs, G_A is usually parameterized by the dipole formula:

$$G_A(Q^2) = \frac{g_A}{(1 + \frac{Q^2}{M_A^2})^2}. \quad (8)$$

Here $g_A = 1.2701 \pm 0.0025$ is the axial constant, known from the neutron β -decay data and M_A is a parameter – the "axial mass". At present, experiments on measurements of the CCQE cross section, performed at different neutrino energies and on different nuclear targets suggest different values for M_A [4]:

<i>d</i> or <i>H</i> – <i>target</i>	$M_A = 1.03 \pm 0.02 \text{ GeV}$	
<i>Fe</i> – <i>target</i>	$M_A = 1.26_{-0.10}^{+0.12+0.08} \text{ GeV},$	<i>MINOS</i>
<i>H₂O</i> – <i>target</i>	$M_A = 1.20 \pm 0.12 \text{ GeV},$	<i>K2K</i>
<i>C</i> – <i>target</i>	$M_A = 1.05 \pm 0.02 \pm 0.06 \text{ GeV},$	<i>NOMAD</i>
<i>C</i> – <i>target</i>	$M_A = 1.35 \pm 0.17 \text{ GeV},$	<i>MiniBooNE</i>

(9)

Though compatible within 2σ errors, these results show a clear discrepancy for the central values of M_A , that could originate in different reasons. The precise determination of the axial FF is important not only for understanding the nucleon structure, but it is a basic ingredient in interpretation of the neutrino oscillation experiments. Here we suggest that measurement of the final nucleon polarization could provide an important independent information about G_A .

3 Polarization of the final nucleon

T-invariance implies that the polarization vector of the final nucleons in (4) lays in the scattering plane. We define its longitudinal s_{\parallel} and transverse s_{\perp} components:

$$\vec{s} = s_{\perp} \vec{e}_{\perp} + s_{\parallel} \vec{e}_{\parallel}, \quad (10)$$

where \vec{e}_{\perp} and \vec{e}_{\parallel} are two orthogonal unit vectors in the scattering plane, $\vec{e}_{\parallel} = \vec{p}'/|\vec{p}'|$, p' is the 4-momentum of the final nucleon. We obtain:

- The transverse polarization exhibits a simple linear dependence on G_A :

$$(J_0 s_{\perp})^{\nu, \bar{\nu}} = \frac{-2E' \sin \theta}{|\vec{q}|} [\pm y G_M^{CC} + (2-y)G_A] G_E^{CC} \quad (11)$$

- The longitudinal polarization s_{\parallel} is expressed solely in terms of G_A and G_M^{CC} , i.e. the best known magnetic form factors of the proton and neutron, the poorly known G_E^{CC} does not enter:

$$(J_0 s_{\parallel})^{\nu, \bar{\nu}} = -\frac{q_0}{|\vec{q}|} [\pm y G_M^{CC} + (2-y) G_A] \left[(2-y) G_M^{CC} \pm \left(y + \frac{2M}{E} \right) G_A \right]. \quad (12)$$

- If the neutrino detector is in a magnetic field, then both s_{\perp} and s_{\parallel} could be measured (like in elastic $e-p$ scattering). Their ratio exhibits a simple linear dependence on G_A :

$$\left(\frac{s_{\parallel}}{s_{\perp}} \right)^{\nu, \bar{\nu}} = \frac{q_0}{2E' \sin \theta} \frac{[(2-y) G_M^{CC} \pm G_A(y + 2M/E)]}{G_E^{CC}}. \quad (13)$$

- The quantity $J_0^{\nu, \bar{\nu}}$ is determined via the differential cross section:

$$J_0^{\nu, \bar{\nu}} = \frac{d\sigma^{\nu, \bar{\nu}}}{dQ^2} \cdot \frac{4\pi}{G_F^2}, \quad (14)$$

and is given by the expression:

$$\begin{aligned} J_0^{\nu, \bar{\nu}} = & 2(1-y) \left[G_A^2 + \frac{\tau(G_M^{CC})^2 + (G_E^{CC})^2}{1+\tau} \right] + \frac{My}{E} \left[G_A^2 - \frac{\tau(G_M^{CC})^2 + (G_E^{CC})^2}{1+\tau} \right] \\ & + y^2 (G_M^{CC} \mp G_A)^2 \pm 4y G_M^{CC} G_A. \end{aligned} \quad (15)$$

Here y , q_0 , $|\vec{q}|$ are kinematic factors, E' is the energy of the final lepton.

4 Numerical results

Using the commonly used parametrizations for the e.m. FFs, we examined the sensitivity of s_{\parallel} and s_{\perp} , and their ratio s_{\parallel}/s_{\perp} on the axial mass for the following values of M_A :

- 1) $M_A = 1.016$ – full (black) line
 - 2) $M_A = 1.20$ – dashed (red) line
 - 3) $M_A = 1.35$ – dash – dotted (blue) line
- (16)

We compared it to the sensitivity of the cross section.

Fig. (1) shows that there is a clear sensitivity in the polarization of the final neutron in $\bar{\nu}_{\mu} + p \rightarrow \mu^+ + n$. It is most clearly pronounced for s_{\parallel} and, respectively, for the ratio s_{\parallel}/s_{\perp} . An advantage of s_{\parallel}/s_{\perp} is that many of the systematic uncertainties and radiative corrections cancel, however a magnetic field should be applied to the detector in order to measure s_{\parallel} . This sensitivity holds also for higher values of neutrino energies E . In contrast, Fig. (2) shows that the cross section exhibits very weak sensitivity to M_A .

There is almost no sensitivity to the polarization of the proton in $\nu_{\mu} + n \rightarrow \mu^- + p$, but the polarizations are big and could present an independent measurement of G_A .

References

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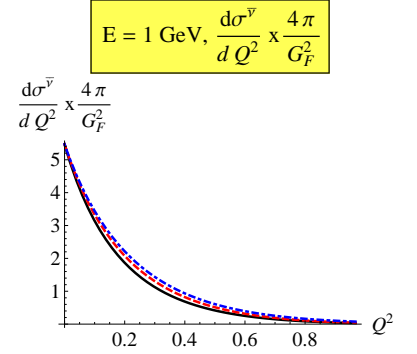


Figure 2: The dependence on M_A (see eq. (16)) of the cross section of $\bar{\nu}_{\mu} + p \rightarrow \mu^+ + n$ at $E=1$ GeV.

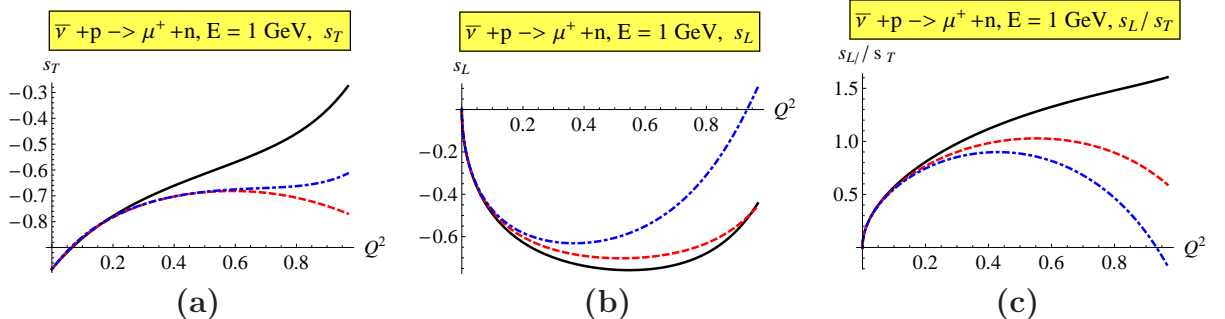


Figure 1: The dependence of the transverse s_T and longitudinal s_L polarizations of the neutron at $E=1$ GeV ((a) and (b)), and their ratio s_L/s_T (c) on the values of M_A (eq. (16)) in $\bar{\nu}_{\mu} + p \rightarrow \mu^+ + n$.